

Rayleigh-Taylor Instability of Viscous-Viscoelastic Fluids in Presence of Suspended Particles Through Porous Medium

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The Rayleigh-Taylor instability of a Newtonian viscous fluid overlying an Oldroydian viscoelastic fluid containing suspended particles in a porous medium is considered. As in both Newtonian viscous-viscous fluids the system is stable in the potentially stable case and unstable in the potentially unstable case, this holds for the present problem also. The effects of a variable horizontal magnetic field and a uniform rotation are also considered. The presence of magnetic field stabilizes a certain wave-number band, whereas the system is unstable for all wave-numbers in the absence of the magnetic field for the potentially unstable configuration. However, the system is stable in the potentially stable case and unstable in the potentially unstable case for highly viscous fluids in the presence of a uniform rotation.

1. Introduction

A comprehensive account of the instability of a plane interface between two Newtonian fluids, under various assumptions of hydrodynamics and hydro-magnetics, has been given by Chandrasekhar [1]. Bhatia [2] has considered the Rayleigh-Taylor instability of two superposed viscous conducting fluids in the presence of a uniform horizontal magnetic field. Sharma [3] has studied the instability of the plane interface between two superposed Oldroydian viscoelastic conducting fluids in the presence of a uniform magnetic field. Bhatia and Steiner [4] have studied the thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation and have found that the rotation has a destabilizing effect, in contrast to its stabilizing effect on a Newtonian fluid. Eltayeb [5] has studied the convective instability in a rapidly rotating Oldroydian viscoelastic fluid.

In geophysical situations the fluid is often not pure but contains suspended particles. Scanlon and Segel [6] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number is reduced because of the heat capacity of the particles. The suspended particles were thus found to destabilize the layer. The medium has been considered to be non-porous in all the above studies.

The flow through porous media is of considerable interest for petroleum engineers and in geophysical

fluid dynamicists. Darcy's equation is a macroscopic equation which describes the flow of an incompressible Newtonian fluid of viscosity μ through a homogeneous and isotropic porous medium of permeability k_1 . In this equation the usual viscous term is replaced by the resistance term $-(\mu/k_1)\mathbf{v}$, where \mathbf{v} is the filter velocity of the fluid. The thermal instability of fluids in a porous medium in the presence of suspended particles has been studied by Sharma and Sharma [7]. The suspended particles and the permeability of the medium were found to destabilize the layer. The Rayleigh instability of a thermal boundary layer in the flow through a porous medium has been considered by Wooding [8]. Oldroyd [9] proposed a theoretical model for a class of viscoelastic fluids. An experimental demonstration by Toms and Strawbridge [10] revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd.

The instability in a porous medium of a plane interface between viscous and viscoelastic fluids containing suspended particles may be of interest in geophysics and biomechanics and is therefore studied in the present paper. The effects of a variable horizontal magnetic field and uniform rotation, bearing relevancy in geophysics, are also considered.

2. Perturbation Equations

Let T_{ij} , τ_{ij} , e_{ij} , μ , λ , $\lambda_0 (< \lambda)$, p , δ_{ij} , v_i , α_i , and d/dt denote respectively the total stress tensor, the shear

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stress tensor, the rate-of-strain tensor, the viscosity, the stress relaxation time, the strain retardation time, the isotropic pressure, the Kronecker delta, the velocity vector, the position vector and the mobile operator. Then the Oldroydian viscoelastic fluid is described by the constitutive relations

$$\begin{aligned} T_{ij} &= -p \delta_{ij} + \tau_{ij}, \\ \left(1 + \lambda \frac{d}{dt}\right) \tau_{ij} &= 2 \mu \left(1 + \lambda_0 \frac{d}{dt}\right) e_{ij}, \\ e_{ij} &= \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right). \end{aligned} \quad (1)$$

Relations of the type (1) were proposed and studied by Oldroyd [9]. Oldroyd showed that many rheological equations of general validity reduce to (1) when linearized. $\lambda_0 = 0$ yields the Maxwellian fluid, whereas $\lambda = \lambda_0 = 0$ gives the Newtonian viscous fluid.

Consider a static state in which an incompressible Oldroydian viscoelastic fluid containing suspended particles is arranged in horizontal strata in a porous medium. The character of the equilibrium of this initial static state is determined, as usual, by supposing that the system is slightly disturbed and by following its further evolution.

Let $\mathbf{v}(u, v, w)$, ϱ and p denote respectively the velocity of pure fluid, the density and the pressure; $\mathbf{u}(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the velocity and number density of the suspended particles, respectively. $K = 6\pi\varrho v\eta$, where η is the particle radius, v is the Stokes' drag coefficient, $\mathbf{u} = (l, r, s)$, $\bar{x} = (x, y, z)$ and $\lambda' = (0, 0, 1)$. Let ε , k_1 , and g stand for medium porosity, medium permeability and acceleration due to gravity, respectively. Then the equations of motion and continuity for the Oldroydian viscoelastic fluid containing suspended particles in a porous medium are

$$\begin{aligned} \frac{\varrho}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t}\right) \left[\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{v} \cdot \nabla) \mathbf{v} \right] \\ = \left(1 + \lambda \frac{\partial}{\partial t}\right) \left[-\nabla p - \varrho g \lambda' + \frac{KN}{\varepsilon} (\mathbf{u} - \mathbf{v}) \right] \\ - \left(1 + \lambda_0 \frac{\partial}{\partial t}\right) \frac{\mu}{k_1} \mathbf{v}. \end{aligned} \quad (2)$$

$$\nabla \cdot \mathbf{v} = 0. \quad (3)$$

Since the density of the moving fluid remains unchanged, we have

$$\varepsilon \frac{\partial \varrho}{\partial t} + (\mathbf{v} \cdot \nabla) \varrho = 0. \quad (4)$$

In the equations of motion (2), by assuming a uniform spherical particle shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term proportional to the velocity difference between the particles and the fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion of the particles. The distances between particles are assumed quite large compared with their diameter, so that interparticle reactions are ignored. The effects of pressure, gravity and Darcian force on the suspended particles are negligibly small and therefore ignored. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$mN \left[\frac{\partial \mathbf{u}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = K N (\mathbf{v} - \mathbf{u}), \quad (5)$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla \cdot (N \mathbf{u}) = 0. \quad (6)$$

Let $\mathbf{v}(u, v, w)$, $\delta \varrho$, δp and $\mathbf{u}(l, r, s)$ denote respectively the perturbations in fluid velocity $(0, 0, 0)$, fluid density ϱ , fluid pressure p and particle velocity $(0, 0, 0)$. Then the linearized perturbation equations of the fluid-particle layer are

$$\begin{aligned} \frac{\varrho}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial \mathbf{v}}{\partial t} &= \left(1 + \lambda \frac{\partial}{\partial t}\right) \\ &\cdot \left[-\nabla \delta p + g \delta \varrho + \frac{KN}{\varepsilon} (\mathbf{u} - \mathbf{v}) \right] \\ &- \left(1 + \lambda_0 \frac{\partial}{\partial t}\right) \frac{\mu}{k_1} \mathbf{v}, \end{aligned} \quad (7)$$

$$\nabla \cdot \mathbf{v} = 0, \quad (8)$$

$$\varepsilon \frac{\partial}{\partial t} \delta \varrho = -w(D\varrho), \quad (9)$$

$$\left(\frac{m}{K} \frac{\partial}{\partial t} + 1 \right) \mathbf{u} = \mathbf{v}, \quad (10)$$

and

$$\frac{\partial M}{\partial t} + \nabla \cdot \mathbf{u} = 0, \quad (11)$$

where $M = \frac{\varepsilon N}{N_0}$ and N_0 , N stand for initial uniform number density and perturbation in number density,

respectively, $g(0, 0, -g)$ is the acceleration due to gravity and $D = \frac{d}{dz}$.

Analysing the disturbances into normal modes, we seek solutions whose dependence on x , y , and t is given by

$$\exp(i k_x x + i k_y y + n t), \quad (12)$$

where k_x, k_y are horizontal wave numbers, $k^2 = k_x^2 + k_y^2$, and n is a complex constant.

For a perturbation of the form (12), (7)–(10) give after eliminating u ,

$$\begin{aligned} \frac{1}{\varepsilon} \left[\varrho + \frac{m N}{(\tau n + 1)} \right] (1 + \lambda n) n u \\ = (1 + \lambda n) (-i k_x \delta p) - (1 + \lambda_0 n) \frac{\mu}{k_1} u, \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{1}{\varepsilon} \left[\varrho + \frac{m N}{(\tau n + 1)} \right] (1 + \lambda n) n v \\ = (1 + \lambda n) (-i k_y \delta p) - (1 + \lambda_0 n) \frac{\mu}{k_1} v, \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{1}{\varepsilon} \left[\varrho + \frac{m N}{(\tau n + 1)} \right] (1 + \lambda n) n w \\ = (1 + \lambda n) [-D \delta p - g \delta \varrho] - (1 + \lambda_0 n) \frac{\mu}{k_1} w, \end{aligned} \quad (15)$$

$$i k_x u + i k_y v + D w = 0, \quad (16)$$

$$\varepsilon n \delta \varrho = -w(D \varrho), \quad (17)$$

where $\tau = m/K$.

Eliminating δp between (13)–(15) and using (16) and (17), we obtain

$$\begin{aligned} \frac{n}{\varepsilon} (1 + \lambda n) [D(\varrho D w) - k^2 \varrho w] + \frac{n(1 + \lambda n)}{\varepsilon(\tau n + 1)} \\ \cdot [D(m N D w) - k^2 m N w] + \frac{(1 + \lambda_0 n)}{k_1} \\ \cdot [D(\mu D w) - k^2 \mu w] + \frac{(1 + \lambda n)}{\varepsilon n} g k^2 (D \varrho) w = 0. \end{aligned} \quad (18)$$

3. Two Uniform Viscous and Viscoelastic Fluids Separated by a Horizontal Boundary

Consider the case of two uniform fluids of densities, viscosities, suspended particles number-densities; ϱ_2, μ_2, N_2 (upper Newtonian fluid) and ϱ_1, μ_1, N_1 (lower

Oldroydian viscoelastic fluid) separated by a horizontal boundary at $z=0$. Then, in each region of constant ϱ , constant μ and constant $m N$, (18) reduces to

$$(D^2 - k^2) w = 0. \quad (19)$$

The general solution of (19) is

$$w = A e^{+kz} + B e^{-kz}, \quad (20)$$

where A and B are arbitrary constants.

The boundary conditions to be satisfied in the present problem are:

- (i) The velocity w should vanish when $z \rightarrow +\infty$ (for the upper fluid) and $z \rightarrow -\infty$ (for the lower fluid).
- (ii) $w(z)$ is continuous at $z=0$.
- (iii) The jump condition at the interface $z=0$ between the fluids is obtained by integrating (18) over an infinitesimal element of z including 0 , and is

$$\begin{aligned} \frac{n}{\varepsilon} [\varrho_2 D w_2 - (1 + \lambda n) \varrho_1 D w_1]_{z=0} \\ + \frac{n m}{\varepsilon(\tau n + 1)} [N_2 D w_2 - (1 + \lambda n) N_1 D w_1]_{z=0} \\ + \frac{1}{k_1} [\mu_2 D w_2 - (1 + \lambda_0 n) \mu_1 D w_1]_{z=0} \\ = -\frac{g k^2}{\varepsilon n} [\varrho_2 - (1 + \lambda n) \varrho_1] w_0, \end{aligned} \quad (21)$$

Remember that upper fluid is Newtonian and lower Oldroydian viscoelastic. w_0 is the common value of w at $z=0$.

Applying the boundary conditions (i) and (ii), we can write

$$w_1 = A e^{+kz} \quad (z < 0), \quad (22)$$

$$w_2 = A e^{-kz} \quad (z > 0), \quad (23)$$

where the same constant A has been chosen to ensure the continuity of w at $z=0$. Applying the condition (21) to the solutions (22) and (23), we obtain

$$\begin{aligned} [\tau \lambda \varrho_1] n^4 + [\tau(\varrho_2 + \varrho_1) + \lambda \varrho_1 + N_1 \lambda m + \frac{\varepsilon}{k_1} \tau \lambda_0 \mu_1] n^3 \\ + \left[(\varrho_2 + \varrho_1) + m(N_2 + N_1) + \frac{\varepsilon}{k_1} \tau(\mu_2 + \mu_1) \right. \\ \left. + \frac{\varepsilon}{k_1} \lambda_0 \mu_1 + g k \tau \varrho_1 \lambda \right] n^2 \\ + \left[\frac{\varepsilon}{k_1} (\mu_2 + \mu_1) + g k \varrho_1 \lambda - g k \tau (\varrho_2 - \varrho_1) \right] n \\ - [g k (\varrho_2 - \varrho_1)] = 0. \end{aligned} \quad (24)$$

If $q_2 < q_1$, (24) does not admit of any positive root, and so the system is stable. If $q_2 > q_1$, (24) allows one positive root of n and so the system is unstable.

4. Effect of a Variable Horizontal Magnetic Field

Consider the motion of incompressible, infinitely conducting Newtonian and Oldroydian viscoelastic fluids in a porous medium in the presence of suspended particles and a variable horizontal magnetic field $\mathbf{H}(H(z), 0, 0)$. Let $\mathbf{h}(h_x, h_y, h_z)$ denote the perturbation in the magnetic field, then the linearized perturbation equations are

$$\begin{aligned} \frac{q}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial \mathbf{v}}{\partial t} = & \left(1 + \lambda \frac{\partial}{\partial t} \right) \cdot \left[-\nabla \delta p + g \delta q + \frac{K N}{\varepsilon} (\mathbf{u} - \mathbf{v}) \right. \\ & \left. + \frac{\mu_e}{4\pi} \{ (\nabla \times \mathbf{H}) \times \mathbf{h} + (\nabla \times \mathbf{h}) \times \mathbf{H} \} \right] \\ & - \left(1 + \lambda_0 \frac{\partial}{\partial t} \right) \frac{\mu}{k_1} \mathbf{v}, \end{aligned} \quad (25)$$

$$\nabla \cdot \mathbf{h} = 0, \quad (26)$$

$$\varepsilon \frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{H}) \quad (27)$$

together with (8)–(10). Assume that the perturbation $\mathbf{h}(h_x, h_y, h_z)$ in the magnetic field has also a space and time dependence of the form (12). μ_e stands for the magnetic permeability. Following the procedure as in Sect. 3, we obtain

$$\begin{aligned} & [\tau \lambda q_1] n^4 + [\tau(q_2 + q_1) + \lambda q_1 + N_1 \lambda m + \frac{\varepsilon}{k_1} \tau \lambda_0 \mu_1] n^3 \\ & + \left[(q_2 + q_1) + m(N_2 + N_1) + \frac{\varepsilon}{k_1} \tau (\mu_2 + \mu_1) \right. \\ & \quad \left. + \frac{\varepsilon}{k_1} \lambda_0 \mu_1 + g k \tau q_1 \lambda + k_x^2 v_A^2 \tau \lambda q_1 \right] n^2 \\ & + \left[\frac{\varepsilon}{k_1} (\mu_2 + \mu_1) + g k q_1 \lambda + k_x^2 v_A^2 \lambda q_1 \right. \\ & \quad \left. + k_x^2 v_A^2 \tau (q_2 + q_1) - g k \tau (q_2 - q_1) \right] n \\ & + [k_x^2 v_A^2 (q_2 + q_1) - g k (q_2 - q_1)] = 0, \end{aligned} \quad (28)$$

where, for the sake of simplicity, we have considered that the Alfvén velocities of the two fluids are the same, so that

$$v_A^2 = \frac{\mu_e H_1^2}{4\pi q_1} = \frac{\mu_e H_2^2}{4\pi q_2}.$$

For the potentially stable arrangement $q_2 < q_1$, (28) does not allow any positive root as there is no change of sign. The system is therefore stable. Thus when the ordinary (Newtonian) viscous fluid overlies an Oldroydian viscoelastic fluid in a porous medium in the presence of suspended particles and a variable horizontal magnetic field, the system is stable for the potentially stable configuration.

For the potentially unstable configuration $q_2 > q_1$, if

$$k_x^2 v_A^2 (q_2 + q_1) > g k (q_2 - q_1), \quad (29)$$

(28) does not admit any change of sign and so has no positive root. Therefore the system is stable.

However, if

$$k_x^2 v_A^2 (q_2 + q_1) < g k (q_2 - q_1), \quad (30)$$

the constant term in (28) is negative. Equation (28), therefore, allows one change of sign and so has one positive root. The occurrence of a positive root implies that the system is unstable.

Thus for the unstable case $q_2 > q_1$, the system is stable or unstable according as $k_x^2 v_A^2 (q_2 + q_1)$ is greater than or smaller than $g k (q_2 - q_1)$. In the absence of a magnetic field, (28) has one positive root, and so the system is unstable for all wave numbers for the potentially unstable case. But the magnetic field has got a stabilizing effect and completely stabilizes the wave number band $k > k^*$, where

$$k^* = \frac{g(q_2 - q_1)}{(q_2 + q_1) v_A^2} \sec^2 \theta, \quad (31)$$

and θ is the inclination of the wave vector \mathbf{k} to the direction of the magnetic field \mathbf{H} i.e. $k_x = k \cos \theta$.

5. Effect of Uniform Rotation

Here we consider the motion of an incompressible Oldroydian viscoelastic fluid containing of suspended particles in a porous medium in uniform rotation $\boldsymbol{\Omega}(0, 0, \Omega)$. Then the linearized perturbation equations are

$$\frac{\rho}{\varepsilon} \left(1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial \mathbf{v}}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t} \right) \left[-\nabla \delta p + \mathbf{g} \delta \varrho + \frac{K N}{\varepsilon} (\mathbf{u} - \mathbf{v}) + \frac{2\rho}{\varepsilon} (\mathbf{v} \times \boldsymbol{\Omega}) \right] - \left(1 + \lambda_0 \frac{\partial}{\partial t} \right) \frac{\mu}{k_1} \mathbf{v} \quad (32)$$

together with (8)–(10).

Following the same procedure as in Sect. 3 (and Chandrasekhar [1], p. 443), we obtain

$$1 + \frac{mn(1 + \lambda n) N_1 + mn N_2}{(\tau n + 1) \left[\left\{ (1 + \lambda n) n + (1 + \lambda_0 n) \frac{\varepsilon v}{k_1} \right\} \varrho_1 + \left\{ n + \frac{\varepsilon v}{k_1} \right\} \varrho_2 \right]} - \frac{g k^2 \varrho_2 - g k^2 (1 + \lambda n) \varrho_1}{n \kappa \left[\left\{ (1 + \lambda n) n + (1 + \lambda_0 n) \frac{\varepsilon v}{k_1} \right\} \varrho_1 + \left\{ n + \frac{\varepsilon v}{k_1} \right\} \varrho_2 \right]} + \frac{4(\tau n + 1) \Omega^2 \varrho_2 - 4(1 + \lambda n)^2 (\tau n + 1) \Omega^2 \varrho_1}{\left[(1 + \lambda n) (\tau n + 1) n + (1 + \lambda n) \frac{mn N}{\varrho} + (1 + \lambda_0 n) (\tau n + 1) \frac{\varepsilon v}{k_1} \right] \left[\left\{ (1 + \lambda n) n + (1 + \lambda_0 n) \frac{\varepsilon v}{k_1} \right\} \varrho_1 + \left\{ n + \frac{\varepsilon v}{k_1} \right\} \varrho_2 \right]} = 0, \quad (33)$$

where

$$\kappa = \frac{k}{\left[1 + \frac{1}{2} \frac{4 \Omega^2 (1 + \lambda n)^2 (\tau n + 1)^2}{n(1 + \lambda n) (\tau n + 1) + (1 + \lambda n) \frac{mn N}{\varrho} + (1 + \lambda_0 n) (\tau n + 1) \frac{\varepsilon v}{k_1}} \right]^2} \quad (34)$$

for a highly viscous fluid. $v (= \mu/\rho)$ stands for the kinematic viscosity.

Here we assumed the kinematic viscosities of both fluids to be equal, i.e. $v_1 = v_2 = v$ (Chandrasekhar [1], p. 443) and $\frac{m N}{\varrho} = \frac{m N_1}{\varrho_1} = \frac{m N_2}{\varrho_2}$, as these simplifying assumptions do not obscure any of the essential features of the problem.

Equation (33), after substituting the value of κ from (34) and simplification, yields

$$A_{13} n^{13} + A_{12} n^{12} + A_{11} n^{11} + \dots + A_2 n^2 + A_1 n + A_0 = 0 \quad (35)$$

where

$$A_{13} = \lambda^4 \tau^4 \varrho_1,$$

$$A_0 = -g k (\varrho_2 - \varrho_1) \frac{\varepsilon v}{k_1} \left[\frac{\varepsilon^2 v^2}{k_1^2} + 2 \Omega^2 \right], \quad (36)$$

and the coefficients $A_1 - A_{12}$, being quite lengthy and not needed in the discussion of stability, have not been written here.

For the potentially stable arrangement $\varrho_2 < \varrho_1$, all the coefficients of (35) are positive. So, all the roots of

(35) are either real and negative, or there are complex roots (which occur in pairs) with negative real parts and the rest negative real roots. The system is therefore stable in each case.

For the potentially unstable arrangement $\varrho_2 > \varrho_1$, the constant term is negative, and so there is at least one change of sign in (35). Therefore (35) allows at least one positive root of n , meaning thereby instability of the system.

Thus the effect of uniform rotation on the motion of an incompressible viscous fluid overlying an Oldroydian viscoelastic fluid through a porous medium in the presence of suspended particles makes the system stable for potentially stable cases and unstable for potentially unstable cases.

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